Exam 200[illegible]

Qu 1 a) On pose $\cancel{g(x) = f(x) - 1 = x^3 + x^7 - 1}$ Methode de
$\cancel{\text{on cherche } g(x) = 0}$

) Donnée : f, x_0, N (nombre de points de support / max d'iteration), ε, y

Resultat : x_m

calculer $f'(x) = \frac{df}{dx}(x) \approx \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

Debut

$\cancel{x_1 \leftarrow x_0}$; $x_1 \leftarrow x_0 + 2\varepsilon$; ~~[illegible]~~

tant que $|x_1 - x_0| > \varepsilon$ et $N > 0$ faire

$x_0 \leftarrow x_1$; $f_0 \leftarrow f(x_0)$; $f'_0 \leftarrow f'(x_0)$;

$x_1 \leftarrow x_0 + (y - f_0) \frac{1}{f'_0}$;

$N = N - 1$;

Fin

si $N < 0$ alors $x_1 \leftarrow$ "echec"

Fin

a) Dans notre cas $f(x) = x^3 + x^7$

on cherche $\tilde{x}$ tel que $f(\tilde{x}) = 1$ on pose $y = 1 = f(\tilde{x})$

$$f(\tilde{x}) = f(x_0 + (x - x_0)) = f(x_0) + f'(x_0)(x - x_0) + \dots$$

$$y = f(x_0) + (\tilde{x} - x_0) f'(x_0)$$

$$\tilde{x} = x_0 + \frac{y - f(x_0)}{f'(x_0)}$$

1^a iteration — On prend $x_0 = 1$

$f(x) = x^3 + x^7$ — $f(x_0) = 2$

$f'(x) = 3x^2 + 7x^6$ — $f'(x_0) = 10$

$$x_1 = 1 + \frac{1 - 2}{10} = 1 + \frac{1}{5} = 0,8$$

Qu 2 Methode de Runge Kutta — Resoudre $\frac{dx}{dt}(t) = f(t, x(t))$

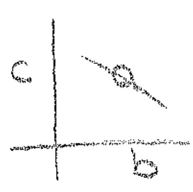
d) ordre 4

$x_{01} = x_0$ — $t_{01} = t_0$

$x_{02} = x_0 + \frac{1}{2} f(t_{01}, x_{01}) h$ — $t_{02} = t_0 + \frac{1}{2} h$

$x_{03} = x_0 + \frac{1}{2} f(t_{02}\ x_{02}) h$ — $t_{03} = t_0 + \frac{1}{2} h$

$x_{04} = x_0 + f(t_{03}\ x_{03}) h$ — $t_{04} = t_0 + h$



$$x_1 = x_0 + h \left[\frac{1}{6} f(t_{01}, x_{01}) + \frac{2}{6} f(t_{02}\ x_{02}) + \frac{2}{6} f(t_{03}\ x_{03}) + \frac{1}{6} f(t_{04}\ x_{04}) \right.$$

A N

$x_{01} = x_0$ — $t_{01} = 0$

$x_{02} = x_0 + \frac{h}{2}(t_{01} + x_0^2) = x_0 \left(1 + \frac{h x_0}{2}\right)$ — $t_{02} = h/2$

$x_{03} = x_0 + \frac{h}{2}\left(\frac{h}{2} + x_0^2 \left(1 + \frac{h x_0}{2}\right)^2\right)$ — $t_{03} = h/2$

$x_{04} = x_0 + h\left(\frac{h}{2} + \left[\frac{h}{2} + x_0^2 \left(1 + \frac{h x_0}{2}\right)^2\right]^2\right)$ — $t_{04} = h$